Analytical Model and Finite-Element Model of an Inductive Displacement Sensor With a Highly Conductive Marker

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The displacement of a metal sphere causes a change of inductance in a measurement coil. This effect is adequately represented by a finite-element model as well as by a semi-analytical formula, both validated by measurements. The displacement sensor is implanted in an artificial lens for cataract patients.

***Index Terms*—Analytical models, Inductance measurement, Magnetic sensors, Numerical simulation.**

I. INTRODUCTION

THE contraction of the ciliary muscle controls the focus of the eye. An implanted metal spherical marker causes a change of the inductance of a coil placed in the eye's lens (Fig. 1) [1], [2], [3]. The interaction between the highly conductive marker and the coil is highly complicated [4], [5]. This paper develops a cheap model to be implemented in the electronic control scheme.

II. APPROXIMATE MODEL

According to the applied frequencies (a few MHz) and the electromagnetic properties of human tissue and metal, wave lengths of several meters are expected in tissue and a skin depth of a few micrometer is expected in the metal sphere. This motivates the use of the magnetoquasistatic approach with the marker modelled as perfectly electric conducting (PEC) boundary [6], [7]. The marker causes a partial shielding of the magnetic field. The impact of the marker is assumed to be proportional to the magnetic energy W_d expelled by the marker's volume. In an axial configuration (Fig. 2) and using Biot-Savart's law [8], the relative change of inductance can be approximated by [2]

$$\frac{\Delta L}{L_*} = \frac{\Delta W}{W_*} = \frac{2}{3}\frac{\mathcal{R}^3}{(\mathcal{R}^2+\delta^2)^3}. \tag{1}$$

Here, $\mathcal{R} = \frac{R}{r}$ is the coil radius and $\delta = \frac{d}{r}$ is the displacement, both relative to the marker radius, L_* and W_* are the inductance and the magnetic energy in the situation without marker and ΔL and $\Delta W = W_* - W_d$ are the changes thereof due to the marker's presence. Model 1 is sufficiently cheap and allows inclusion in a simple electronic control scheme, but may lack accuracy due to the severe assumptions involved.

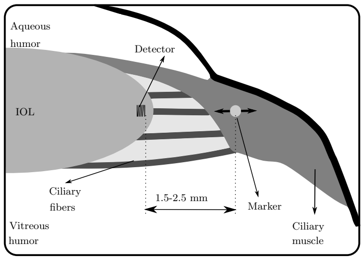

Fig. 1. Marker implanted in the ciliary muscle at a distance between 2 and 4 mm. When the eye accommodates the ciliary muscle moves the marker closer or further away from the sensor (maximum range of 1 mm), causing a change of inductance of the coil [2].

III. FINITE-ELEMENT MODEL

The approximate model is validated by a set of 2D axisymmetric finite-element (FE) models (Fig. 2). The marker is modelled as a PEC boundary. For the considered frequencies, the relative permittivity and permeability of human tissue can be considered as 1 [9]. The coil and sphere are embedded in a sufficiently large computational domain to account for stray fluxes [10]. The FE results are in good agreement with the results of the approximate model (Fig. 3). Remarkable is the fact that also at larger distances ($d > 3$ mm), both approximate model and the FE model feature a sufficient resolution.

The explicit change of the marker's position necessitate remeshing between two successive FE runs. An alternative approach involves a conformal mapping of a cylindrical region between coil and marker with varying height $H(d)$ to a reference cylinder with fixed height $H_{\mathrm{ref}} = H(d_{\mathrm{ref}})$. Then, instead of changing the model geometry and remeshing, the displacement is modelled by a varying, fictitious, anisotropic permeability $(\tilde{\mu}_r, \tilde{\mu}_z) = (\xi\mu_0, \mu_0/\xi)$ with $\xi = d/d_{\mathrm{ref}}$, which is assigned to the reference cylinder region.

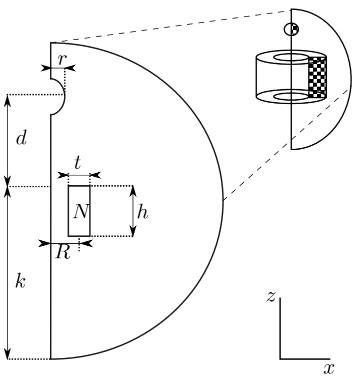

Fig. 2. Axially symmetric 2D model of a coil with $h = 0.42$ mm, thickness $t = 0.21$ mm, radius $R = 1$ mm and number of turns $N = 1$ enclosed in a fixed modelling sphere with radius $k = 20$ mm together with a spherical marker with radius r placed at a distance d from the face of the coil. Enlarging the coil radius by a factor also enlarges t,h and k by the same factor.

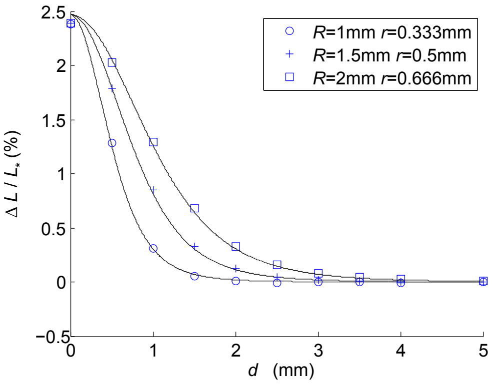

Fig. 3. Simulated data with $\mathcal{R} = 3$ as function of marker distance d plotted together with Eq. 1 for the corresponding coil radius R and marker radius r.

IV. VALIDATION BY MEASUREMENT

An experiment was set up with aluminium spheres of different diameter (Fig. 4). The inductance change is measured as a shift of the resonance frequency of a Colpitts oscillator, which is observed as a change of DC voltage in a phase lock loop [11]. The expected frequency shift is

$$\Delta f = f_* - f_d \simeq -\frac{1}{3}\frac{\mathcal{R}^3}{(\mathcal{R}^2+\delta^2)^3} f_* \tag{2}$$

with f_* the frequency without sphere and f_d the frequency with a sphere at distance d [2]. The measured frequency shift corresponds very well to the frequency shift predicted by (2) (Fig. 5).

V. CONCLUSIONS

An inductive distance measurement system is equipped with a model that relates a change in resonance frequency to the distance between a metal sphere and a measurement coil. The validation by a FE model and verification by measurements show a very good agreement. The technique is mature for inclusion in a artificial lens with accommodation.

VI. ACKNOWLEDGEMENT

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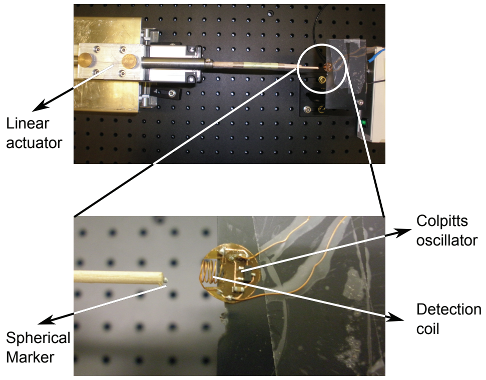

Fig. 4. Experimental setup consisting of a spherical marker placed on a wooden stick, a stepper motor, Colpitts oscillator with detection coil, phase locked loop, oscilloscope and a pc (oscilloscope and pc not depicted here). The stepper motor moves the spherical marker along the central axis of the coil thereby changing the inductance L of the coil.

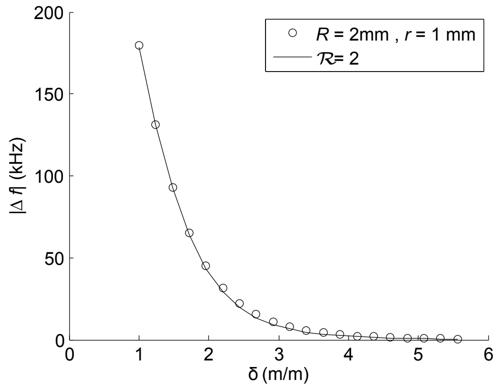

Fig. 5. Experimentally measured frequency shift for $\mathcal{R} = 2$ as function of scaled distance δ plotted together with Eq. 2 (full line).

REFERENCES

[1] D. Doornaert, C. Glorieux, H. De Gersem, R. Puers, W. Spileers, and J. Blanckaert, "Intraocular electro-optic lens with ciliary muscle controlled accommodation," *Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, pp. 3190–3, July 2013.

[2] D. Doornaert, C. Glorieux, R. Puers, H. De Gersem, W. Spileers, and J. Blanckaert, "Physiological constraints for an intraocular inductive distance sensor," in *36th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, Chicago, USA, Aug. 2014, pp. 646–649.

[3] C. Glorieux, J. Blanckaert, and R. Puers, "Bionic eye lens PCT/BE2011/000045," 2006.

[4] A.A. Kolyshkin and R. Vaillancourt, "Impedance of a single-turn coil due to a double-layered sphere with varying properties," *IEEE Transactions on Magnetics*, vol. 31, no. 3, pp. 2274–2279, May 1995.

[5] S. Babic, S. Salon, and C. Akyel, "The mutual inductance of two thin coaxial disk coils in air," *IEEE Transactions on Magnetics*, vol. 40, no. 2, pp. 822–825, Mar. 2004.

[6] H.K. Dirks, "Quasi-stationary fields for microelectronic applications," *Electrical Engineering*, vol. 79, pp. 145–155, 1996.

[7] R.L. Stoll, *The Analysis of Eddy Currents*, Clarendon Press, 1974.

[8] F.W. Grover, *Inductance Calculations*, Dover, 1964.

[9] S. Gabriel, R.W. Lau, and C. Gabriel, "The dielectric properties of biological tissues: II. Measurements in the frequency range 10 Hz to 20 GHz," *Physics in Medicine and Biology*, vol. 41, pp. 2251–2269, 1996.

[10] Q. Chen and A. Konrad, "A review of finite element open boundary techniques for static and quasistatic electromagnetic field problems," *IEEE Transactions on Magnetics*, vol. 33, no. 1, pp. 663–676, Jan. 1997.

[11] Emad Hegazi, Jacob Rael, and Asad Abidi, *The Designer's Guide to High-Purity Oscillators*, Kluwer Academic Publishers, 2004.